SÉRIES TEMPORAIS – EXAME RECURSO – 31/1/2014 – SOLUÇÃO

1

HOLT: $a(t) = \alpha Y_t + (1-\alpha)[a(t-1)+b(t-1)] = a_{t-1}+b_{t-1}+\alpha e_t$ com $e_t = Y_t - a_{t-1} - b_{t-1}$

$b(t) = \beta[a(t)-a(t-1)] + (1-\beta)\, b(t-1) = b_{t-1} + \alpha\beta e_t$ " "

AED: $a(t) = a_{t-1}+b_{t-1}+\alpha(2-\alpha)e_t$ e $b_t = b_{t-1}+\alpha^2 e_t$ (Gardner, 1985)

Fazendo $\alpha=\beta$ (HOLT) sai $b_t = b_{t-1}+\alpha^2 e_t$ (igual ao AED)

2

$Y_t = \phi Y_{t-1} + \varepsilon_t = \phi(\phi Y_{t-2}+\varepsilon_{t-1}) + \varepsilon_t = \dots = (1+\psi_1 B+\psi_2 B^2+\dots)\varepsilon_t$

com $\psi_0=1$, $\psi_1=\phi$, $\psi_2=\phi^2, \dots$

3

a) $cov(W_t, W_{t-k}) = cov(Y_t - Y_{t-1}, Y_{t-k} - Y_{t-k-1}) = cov(Y_t, Y_{t-k}) +$
$cov(Y_{t-k} - Y_{t-k-1}) + cov(-Y_{t-1}, Y_{t-k}) + cov(-Y_{t-1}, -Y_{t-k-1}) =$

$$= \frac{\phi^k - \phi^{k+1} - \phi^{k-1} + \phi^k}{1-\phi^2}\sigma_\varepsilon^2 = \dots = -\left[\frac{1-\phi}{1+\phi}\right]\phi^{k-1}\sigma_\varepsilon^2$$

b) $$Var(W_t) = Var(Y_t - Y_{t-1}) = \frac{2(1-\phi)}{1-\phi^2}\sigma_\varepsilon^2 = \frac{2}{1+\phi}\sigma_\varepsilon^2$$

4

a) $d=1$, $p=3$, $q=0$

b) $P_{2014} = \hat{Y}_{2013}(1) = 1778.12$ $P_{2016} = \hat{Y}_{2013}(3) = 1697.27$

$P_{2015} = \hat{Y}_{2013}(2) = 1719.79$

5

a) $P_{100} = \hat{Y}_{99}(1) = 111.693$; $P_{101} = 111.495$; $P_{102} = 113.203$; $P_{103} = 111.144$

b) $I.P.(t=100) = 111.693 \pm 1.96 \times \sqrt{9}$, $I.P.(t=101) = 111.495 \pm 1.96 \times \sqrt{9(1+(1-1.3)^2)}$

...

6

a) $$Y_t = \left[w_0 + \frac{w_1 B}{1-\delta B} + \frac{w_2}{1-B}\right] P_t^{[T]}$$